

## Applications of Double Integrals: Center of Mass and Surface Area

1. A flat plate (“lamina”) is described by the region  $\mathcal{R}$  bounded by  $y = 0$ ,  $x = 1$ , and  $y = 2x$ . The density of the plate at the point  $(x, y)$  is given by the function  $f(x, y)$ .

(a) Write double integrals giving the first moment of the plate about the  $x$ -axis and the first moment of the plate about the  $y$ -axis. (You need not convert to iterated integrals.)

(b) The center of mass of the plate is defined to be the point  $(\bar{x}, \bar{y})$  where

$$\bar{x} = \frac{\text{first moment of plate about } y\text{-axis}}{\text{mass of plate}} \quad \text{and} \quad \bar{y} = \frac{\text{first moment of plate about } x\text{-axis}}{\text{mass of plate}}.$$

Write expressions for  $\bar{x}$  and  $\bar{y}$  in terms of iterated integrals.

2. In this problem, we will look at the portion of the paraboloid  $z = x^2 + y^2 + 1$  with  $z < 10$ . Let’s call this surface  $\mathcal{S}$ .

(a) Parameterize the surface  $\mathcal{S}$ .<sup>(1)</sup> Describe any restrictions on the parameters.

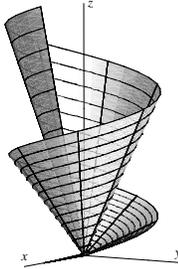
(b) Find the surface area of  $\mathcal{S}$ .

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<sup>(1)</sup>Remember that this basically means we want to describe the surface using two variables — those are the parameters. Although we may use cylindrical or spherical coordinates to come up with a parameterization, our final parameterization should always describe the surface in Cartesian coordinates.

3. In each part, write a double integral that expresses the surface area of the given surface  $\mathcal{S}$ . Sketch the region of integration of your double integral. (You do not need to convert the double integral to an iterated integral or evaluate it.)

(a)  $\mathcal{S}$  is parameterized by  $\vec{r}(u, v) = \langle u \cos v, u \sin v, uv \rangle$ ,  $0 \leq u \leq 2\pi$ ,  $0 \leq v \leq 4\pi$ .



(b)  $\mathcal{S}$  is the part of the surface from (a) under the plane  $z = 20$ .

4. Find the surface area of the following surfaces.

(a)  $\mathcal{S}$  is the portion of the plane  $3x - 3y + z = 12$  which lies inside the cylinder  $x^2 + y^2 = 1$ .

(b)  $\mathcal{S}$  is the portion of the plane  $3x - 3y + z = 12$  which lies inside the cylinder  $y^2 + z^2 = 1$ .

(c)  $\mathcal{S}$  is a sphere of radius 1.